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1 Introduction

A dissipation free inertial range of scale is assumed to exist in a theory of developed turbulence of incompressible fluid. Under the scenario of isotropic homogeneous turbulence the infrared (IR) pumping of energy into this range is executed by the large vortices of scale L; the small scale l_0 defines the bottom of the range, where the dissipation of energy because of viscosity becomes significant.

The attempts to generalize the approach advanced in [1]-[2] on a case of weak compressibility of a fluid was established in [3]-[4]. It was shown, that since some values of Mach number $Ma = v_c/c$ (v_c is the velocity of turbulent pulsation, means the velocity of sound) a new parameter c^{-2} occurs in the inertial range. In the case of large Mach numbers it makes impossible to construct a theory of developed turbulence with the help of dimensionless parameters.

Under such circumstances the most important problem of the theory is to describe the scaling functions. The general method of construction of decomposition of IR asymptotics of Green functions on Ma number (zero order of decomposition is an incompressible fluid) was advanced in [5]. To study the strength of the method we offer the adaptation of it for a model of stochastic magnetic hydrodynamics (MHD) in the frameworks of present paper.

The original results have been obtained by us about the renormalization and critical dimensions of the composite operators (the local averages) of fields in magnetic hydrodynamics are significant because of it is available for an experimental measurement.

2 Model of stochastic magnetic hydrodynamics of compressible fluid

As initial micromodel a system of equations of stochastic magnetic hydrodynamics, completed by the equation of continuity and condition of weak compressibility of a fluid is chosen.

$$\partial_t \varphi = -(\varphi\partial)\varphi + \frac{1}{\rho}\left(\eta\Delta\varphi + \eta'\partial(\partial\varphi) - \partial p - \frac{1}{2}\partial\theta^2 + (\theta\partial)\theta\right) + \mathbf{f}^{\varphi}, \tag{1}$$

$$\partial_t \theta = -(\varphi\partial)\theta + \nu_m\Delta\theta + (\theta\partial)\varphi - \theta(\partial\varphi) + \mathbf{f}^{\theta}, \tag{2}$$

$$\partial_t \rho + \partial(\rho\varphi) = 0, \quad p = c^2\rho. \tag{3}$$

The Navier-Stokes equation (1) for the velocity of fluid $\varphi(\mathbf{x},t)$ contains the terms are responsible for interaction with the pseudovector $\theta(\mathbf{x},t) = \mathbf{B}/\sqrt{4\pi\rho_0}$. We assume here ρ_0 as an equilibrium value of density of fluid (further we accept $\rho_0 = 1$); $p(\mathbf{x},t)$ is a field of pressure, η is a factor of shift viscosity, $\eta' = (\eta/3 + \zeta)$, where ζ is a factor of volumetric viscosity, c- velocity of sound. The magnetic viscosity coefficient $\nu_m = c^2/4\pi\sigma = \lambda\nu$ (here σ means the conductivity of fluid, c is the speed

of light) is useful to be connected with a factor of kinematic viscosity with the help of inverse Prandtl number λ.

For the external random force $\mathbf{f}^{\varphi}$ referred to unit of weight and a rotor of the random current $\mathbf{f}^{\theta}$ it is supposed Gaussian distribution with the correlators $D_{ij}^{\phi\phi}$ [5]:

$$D_{ij}^{\varphi\varphi} = g_{10}\nu_0^3 P_{ij} d_{vv} + c^{-4} g_{40} Q_{ij} d_{uu}. \quad D_{ij}^{\theta\theta} = g_{20}\nu_0^3 P_{ij} d_{\theta\theta}, \quad D_{ij}^{v\theta} = g_{30}\nu_0^3 \varepsilon_{ijm} k_m d_{v\theta},$$
$$d_{vv} = d_{uu} = k^{4-d-2\varepsilon}, \quad d_{\theta\theta} = k^{4-d-2a\varepsilon}, \quad d_{v\theta} = k^{3-d-(1+a\varepsilon)}. \tag{4}$$

(D_{ij} do not depend of frequency), d - dimension of space, (generally speaking the antisymmetric pseudotensor ε_{ism} is determined only at $d = 3$).

The factors g_{i0} in (4) play a role of charges; the positive constant a is an arbitrary parameter of theory. ε is served to construct the decomposition of correlation functions. Really $\varepsilon_p = 2$; it simulates the energy pumping from the large-scale movements of a fluid.

In the framework of approach it is useful to assume $g_3 = \xi\sqrt{g_1 g_2}$. A new dimensionless parameter ξ replacing g_3 is not a charge but an arbitrary ($|\xi| \leq 1$) parameter of the theory.

A generating functional of the correlation functions of the stochastic problem (1-4) is defined by the formula

$$G(A_\phi) = \int \mathcal{D}\Phi \det M \exp(S(\Phi) + \Phi A_\Phi), \quad \Phi = \ \varphi, \varphi', \theta, \theta', p, p' \ ,$$

in which the action functional $S(\Phi)$ is a kind of

$$\begin{aligned} S(\Phi) = {} & 1/2\varphi_i' D_{ij}^{\varphi\varphi}\varphi_j' + 1/2\theta_i' D_{ij}^{\theta\theta}\theta_j' + \varphi_i' D_{ij}^{\varphi\theta}\theta_j' + \varphi_i'\Big[-\partial_t\varphi_i - (\varphi\partial)\varphi_i + \\ & +1/(1+p/c^2)\Big(\eta\Delta\varphi_i + \eta'\partial_i(\partial\varphi) - \partial_i p - 1/2\partial_i\theta^2 + (\theta\partial)\theta_i\Big)\Big] + \theta_i'\Big[-\partial_t\theta_i + \\ & +\lambda\nu\Delta\theta_i - (\varphi\partial)\theta_i + (\theta\partial)\varphi_i - \theta_i(\partial\varphi)\Big] + p'\Big[1/c^2\partial_t p + (\partial u + 1/c^2\partial(p\varphi)\Big]. \end{aligned} \tag{5}$$

Necessary integration on $\mathbf{x}$, t and summation on indices are meant. A_Φ are the sources of appropriate fields; $\det M$ is an unessential factor under conditions of the theory [3].

A perturbation theory for $G(A_\Phi)$ is received under the decomposition of $\exp S$ on the terms of (5) containing the products of three and more fields. $1/(1+p/c^2)$ should be presented as the power series of p/c^2. The correlation functions of an arbitrary number of $\rho \equiv p/c^2$ are divergent in the theory, so the infinite number of counterterms would be required for renormalization of (5). Therefore the application of renormalization group approach (RG) to (5) is impossible.

At $c^{-2} = 0$ integration on the field p' of the generating functional yields $\delta(\partial\varphi)$. It corresponds to the case of incompressible fluids $\rho \to \rho_0$. By the way a field of velocity becomes $v_i = P_{ij}\varphi_j$, $P_{ij} = \delta_{ij} - k_i k_j/k^2$, and the longitudinal component $u_i = Q_{ij}\varphi_j$, $Q_{ij} = \delta_{ij} - P_{ij}$ is excluded from stochastic equations (1-2). The appropriate theory contains the transversal fields only: v, v' and θ, θ'. For elimination of divergences in such theory a final number of counterterms is required only.

The theory of incompressible conductive fluid was investigated in [6] in the first order on ε with the help of recursive renormalization group, and then under the field theory approach in [7]. The scaling behaviour of correlation functions in such model was considered in [8], and the critical dimensions of the composite operators of junior canonical dimensions were calculated in [9].

Hereinafter it will be convenient pursuant to estimation of $u \sim c^{-2}$ [5] to make following replacement of variables in (5):

$$u \to u/c^2, \quad p' \to c^2 p'.$$

The action (5) is invariant to a Galileian transformations of fields:

$$\varphi(\mathbf{x}, t) \to \varphi(\mathbf{x} + \mathbf{a}t, t) - \mathbf{a}, \quad \Phi(\mathbf{x}, t) \to \Phi(\mathbf{x} + \mathbf{a}t, t),$$

where Φ are the other fields, except of φ, which participates in (5) whether under a derivative or in the covariant derivative $\mathcal{D}_t = \partial_t + v\partial$.

3 IR perturbation theory in a kinetic mode of compressible fluid

The action functional (5) develops from the incompressible one $S_{ic}(v, v', \theta, \theta')$ and the terms containing the longitudinal and scalar fields u, u', p, p',. For S_{ic} the IR-behaviour was studied by RG. The RG transformation has two fixed points

$$\beta_{g_\alpha} = 0, \quad \omega_{\alpha\delta} \equiv \partial\beta_{g_\alpha}/\partial g_\delta > 0, \quad \beta_{g_\alpha} \equiv D_M g_\alpha, \quad D_M \equiv M\partial_M,$$

(M is a renormalization mass parameter) determining two different critical regimes of turbulent behaviour [8]. So called "kinetic" regime realizes if a trajectory of invariant charges $\bar{g}_\alpha$ falls into a vicinity of a point

$$g'_{1*} = \frac{g_{1*}}{B\lambda_*} = \frac{\varepsilon(1+\lambda_*)}{15}, \quad g'_{2*} = \frac{g_{2*}}{B\lambda_*^2} = 0, \quad \lambda_* = \frac{\sqrt{43/3} - 1}{2}, \tag{6}$$

in the charge space. $B = d(d+2)(4\pi)^{d/2}\Gamma(d/2)$. The point is IR steady while $a < 1.16$.

A magnetic regime is been steady at $a \geq 0.25$ to be correspond to a point

$$g'_{1*} = \lambda_* = 0, \quad g'_{2*} = a\varepsilon. \tag{7}$$

Because of trivial values of invariant charges under (6) conditions it is useful to apply the new variables [8]:

$$\theta \to \sqrt{g_2\nu^3}M^{a\varepsilon}\theta, \quad \theta' \to \theta'/\sqrt{g_2\nu^3}M^{a\varepsilon}, \quad v \to \sqrt{g_1\nu^3}M^{\varepsilon}v, \quad v' \to v'/\sqrt{g_1\nu^3}M^{\varepsilon} \tag{8}$$

to avoid triviality of the correlation functions in the kinetic regime. The position of the fixed point (6) doesn't change under (8).

After integration of $G(A_\Phi)$ on the transversal fields v, v', θ, θ' one can construct a new perturbation theory on c^{-1} as a kind of decomposition of $\exp(S - S_{ic})$ on the terms containing the longitudinal and scalar fields. The appropriate diagrammatic techniques consists of lines of fields u, u', p, p', and the composite operators of v, v', θ, θ'. The propagators of longitudinal and scalar fields join own vertices or connect to the composite operators of transversal fields.

For summation of IR peculiarities in subgraphs are build of transversal fields one can use the results [5], [9] where critical dimensions of the fields and some composite operators are indicated. Dimensions of a number of composite operators containing the magnetic fields θ and θ', have been unknown and are resulted here for the first time.

One can coordinate the diagrammatic techniques with decomposition on c^{-1} with the help of a change of variables in (5):

$$\tilde{q} = p + g_2\nu^3\theta^2/2 - F_p, \quad \tilde{u}_i = u_i + F_{1i} - F_{2i}q, \tag{9}$$

where $F_p = \nu^3\Delta^{-1}\partial_i\partial_k(g_1 v_i v_k - g_2\theta_i\theta_k)$, $\quad F_{1i} = \Delta^{-1}\partial_i\mathcal{D}_t F_p$, $\quad F_{2i} = \Delta^{-1}\partial_i\mathcal{D}_t$, and Δ^{-1} is the Green function of Laplasian.

A problem of renormalization of operator sequences containing a number of F_p-type operators had been discussed already in [5]. The analysis of them was facilitated by the Galileian invariance of theory, since the noninvariant local operators constructed with participation of field v do not admix to a combination $g_1\nu^3\partial_i\partial_k v_i v_k$ accumulated in the perturbation theory [5].

In our case the set of operators mixed due to renormalization appears to be essentially wider owing to Galileian invariancy of θ-field. Renormalization of such sequences demands the knowledge of scaling dimensions of the local composite operators each, which we have not. Therefore, discussing critical behaviour in MHD we are compelled to be limited to approach of one insertion of the operators of transversal fields.

As a result of transformation (8) one can get:

$$\begin{gathered}
S(\Phi) = 1/2 v'_i d^{vv}_{ij} v'_j + 1/2\theta'_i d^{\theta\theta}_{ij}\theta'_j + \xi v'_i d^{v\theta}_{ij}\theta'_j + 1/2 u'_i D^{uu}_{ij} u'_j + v'_i\Big[-\partial_t v_i - \\
-\nu^{3/2} {g_1}^{1/2}(v\partial)v_i - 1/c^2\left((v\partial)\tilde{u}_i + (\tilde{u}\partial)v_i\right) + 1/(1+\tilde{q}/c^2)\Big(\nu\Delta v_i + \\
+g_2 {g_1}^{-1/2}\nu^3(\theta\partial)\theta_i + 1/c^4\nu'\nu^{-3/2}g_1^{-1/2}\tilde{q}\Delta\tilde{u}_i\Big)\Big] + u'_i[-1/c^2\partial_t\tilde{u}_1/c^2 g_1^{1/2}\nu^{3/2}\cdot \\
\cdot\Big((v\partial)\tilde{u}_i + (\tilde{u}\partial)v_i\Big) - 1/c^4(\tilde{u}\partial)\tilde{u}_i - \partial_i p + 1/(1+\tilde{q}/c^2)(-1/c^2\nu^{3/2}\nu g_1^{1/2}\tilde{q}\Delta v_i + \\
+1/c^2\nu'\Delta\tilde{u}_i - 1/2\cdot 1/c^2\partial_i\tilde{q}^2 - 1/c^2\nu^3 g_2\tilde{q}(1/2\partial_i\theta^2 - (\theta\partial)\theta_i))] + \\
+\theta'_i\Big[-\partial_t\theta_i + \lambda\nu\Delta\theta_i - g_1\nu^{3/2}\left((v\partial)\theta_i + (\theta\partial)v_i\right) + 1/c^2\left((\theta\partial)\tilde{u}_i - \theta_i(\partial\tilde{u} - (\tilde{u}\partial)\theta_i)\right]- \\
-p'\Big[(\partial u + 1/c^2\partial_i(\tilde{q}\tilde{u}_i)\Big], \quad \nu' = (\eta+\eta')/\rho_0.
\end{gathered} \tag{10}$$

Square-law part of the functional defines the propagators of perturbation theory:

$$\begin{gathered}
G^{vv'} = (G^{v'v})^{+} = L_1^{-1}, \quad G^{\theta\theta'} = (G^{\theta'\theta})^{+} = L_2^{-1}, \\
G^{vv} = G^{vv'} d_{vv} G^{v'v}, \quad G^{\theta\theta} = G^{\theta\theta'} d_{\theta\theta} G^{\theta'\theta}, \\
G^{\phi'\phi'} = 0, \quad G^{uu'} = (G^{u'u})^{+} = \omega L_3^{-1}, \quad G^{p'p} = (G^{pp'})^{+} = -(i\omega + \nu' k^2) L_3^{-1}, \\
G^{uu} = G^{uu'} D_{uu} G^{u'u}, \quad G^{pp} = G^{pu'} D_{uu} G^{u'p}, \quad G^{up} = 1/c^2 G^{uu'} D_{uu} G^{u'p}, \\
G^{p'u} = i\mathbf{k}/k^2, \quad G^{u'p} = i\mathbf{k} L_3^{*-1}, \\
L_1 = -i\omega + \nu_0 k^2, \quad L_2 = -i\omega + \lambda_0 \nu_0 k^2,
\end{gathered} \tag{11}$$

The propagators of transversal fields are multiple to P_{ij}; other ones have the factor of Q_{ij}.

The scaling dimensions of operators of transversal fields are listed in table 1. In the second column of (tab. 1) the composite operators of transversal fields are submitted. The longitudinal and scalar fields joined with them are specified in column 4. The degree of c^{-2} at an appropriate vertex is indicated in column 5. The third column of the table gives a scaling dimension of the appropriate composite operator at the physical value of $d = 3, \quad \varepsilon = 2$. In the table we have taken $\omega_{1,2} = \min(\omega_1, \omega_2)$ and $\omega_{1,3} = \min(\omega_1, \omega_3)$. The IR-correction indices (ω_α) are the eigenvalues of a matrix $\omega_{\alpha\delta}$. They are known to be $\omega_\alpha > 0$ to provide the stability of a critical regime.

4 Scaling dimensions of the composite operators of transversal fields

One should note that the contributions of composite operators constructed with participation of fields $(g_2\nu^3)^{1/2}\theta$ to the asymptotics of Green functions in the perturbation theory are proportional to degrees of $\bar{g}_2$. Within the framework of RG the estimation of value of $\bar{g}_2$ in the vicinity of g_{2*} is proven:

$$\bar{g}_2 = a_2 s^{\omega_2}$$

(s is a dimensionless scaling parameter; a_2 is an arbitrary constant). So the index ω_2 have to be accounted at the decision of critical dimension of the operators of type mentioned.

The renormalization of composite operators of transversal fields proceeds in the framework of incompressible theory with the renormalized functional of action

$$\begin{gathered}
S_R = \tfrac{1}{2} v' d_{vv} v' + \tfrac{1}{2} \theta' d_{\theta\theta} \theta' + v' d_{v\theta} \theta' + \\
+ v'[-\partial_t v + Z_1 \nu \Delta v - {g_1}^{1/2} \nu^{3/2} M^{\varepsilon} (v\partial) v + Z_3 g_2 {g_1}^{-1/2} \nu^{3/2} M^{(2a-1)\varepsilon} (\theta\partial)\theta] + \\
+ \theta'[-\partial_t \theta + Z_2 \lambda \nu \Delta \theta - {g_1}^{1/2} \nu^{3/2} M^{\varepsilon} (v\partial)\theta + {g_1}^{1/2} \nu^{3/2} M^{\varepsilon} (\theta\partial) v],
\end{gathered}$$

where the one-loop order of renormalization constants were calculated in [7]:

$$\begin{gathered}
Z_1 = 1 - \tfrac{g_1 d(d-1)}{4B\varepsilon} - \tfrac{g_2(d^2+d-4)}{4Ba\lambda^2\varepsilon}, \quad Z_3 = 1 + \tfrac{g_1}{B\lambda\varepsilon} - \tfrac{g_2}{Ba\lambda^2\varepsilon}, \\
Z_2 = 1 - \tfrac{g_1(d+2)(d-1)}{2B\lambda(\lambda+1)\varepsilon} - \tfrac{g_2(d+2)(d-3)}{2Ba\lambda^2(\lambda+1)\varepsilon}.
\end{gathered}$$

4.1 Not local composite operators

It is known, that the local operators (i.e. respected to a same point) can admix to not local one as the result of renormalization. The operators we are interested in the Galileian invariant operators would be admixed only. All the not local composite operators in the case considered contain the single insertion of F_p-type. They have two external derivative; the admixing local operators should have the same property. As far as $d_\partial = 1$, one can show that all such local operators are unessential in comparison with the not local ones [5]. Therefore it is available to limit the consideration of matrix of renormalization constants by the set of elements which are responsible for mixing of not local operators. The appropriate scaling dimensions are the sums of scaling dimensions of local fragments, derivatives, and Δ^{-1}-operators. As a matter of fact it is sufficient to study the scaling dimensions of the local fragments.

4.2 Local composite operators

The dimension of a tensor $\phi_i\phi_k$ was defined in [9]. In case of small a the operator $\partial_i\partial_k v_i v_k$ is essential, its dimension is known precisely (4/3) at real value of ε. Depending on the value of parameter a the contribution of operator $g_2\partial_i\partial_k\theta_i\theta_k$ can occur essential too ($5\frac{1}{5} - 4a + \omega_2$). We know only one-loop value of index $\omega_2 \simeq 4(a-1.16)$ [7], so it is possible to predict shift of dimension ΔF_p at $a \geq 1.1$, i.e. near the boundary of area of stability of kinetic regime.

The dimension of local fragments located in $N^0 3, 4$ in tab.1 can be determined, having considered result of action of the operations ∂_{g_δ}, $g_\delta = g_1, g_2, \lambda$, on a RG equation for renormalized generating functional of spanned Green functions $W^R(A_\Phi)$:

$$D_{RG}W^R(A_\Phi) = 0, \quad D_{RG} = D_M + \beta_{g_\alpha}\cdot\partial_{g_\alpha} - \gamma_1\cdot D_\nu, \quad \gamma_\alpha = D_M \ln Z_\alpha.$$

The linear combinations of operators $C_\alpha = {\omega_{\delta\alpha}}^{-1}[\partial_{g_\delta}, D_{RG}]W$ have in fixed points (6,7) anomalous dimensions ω_α (square brackets here designate a commutator).

In a fixed point is fair $D_{RG}C_\alpha = \omega_{\alpha\delta}C_\delta$, as far as

$$D_{RG}C_\alpha = \omega^{-1}_{\delta\alpha}D_{RG}(\omega_{\delta\alpha}\cdot\partial_{g_\alpha} - \partial_{g_\delta}\gamma_1\cdot D_\nu)W = \Big(D_{RG}\cdot\partial_{g_\alpha} -$$
$$-\omega^{-1}_{\delta\alpha}D_{RG}\partial_{g_\delta}\gamma_1\cdot D_\nu\Big)W = \Big((\omega_{\delta\alpha}\partial_{g_\delta} - \partial_{g_\delta}\gamma_1\cdot D_\nu) - \omega^{-1}_{\delta\alpha}\cdot\beta_{g_\alpha}\partial_{g_\alpha}\partial_{g_\delta}\gamma_1 D_\nu\Big)W =$$
$$= \omega_{\alpha\delta}C_\delta.$$

In the last equality we allowed, that in the fixed point $\beta_{g_\alpha} = 0$. Eigenvalues ω_α matrixes $\omega_{\alpha\delta}$ are the anomalous dimensions of combinations C_α.

In a vicinity of kinetic point (7) the matrix has a kind of:

$$\omega_{\delta\alpha} = \begin{pmatrix} 3g_1\partial_{g_1}\gamma_1 & 3g_1\partial_{g_2}\gamma_1 & 0 \\ 0 & -2a\varepsilon + 3\gamma_1 - \gamma_3 & 0 \\ 0 & \lambda\partial_{g_2}\gamma_1 & -\lambda\partial_\lambda\gamma_2 \end{pmatrix}.$$

The eigenvalues ω_α of matrix coincide with the diagonal elements.

Taking into account, that in kinetic point it is fair: $\partial_{g_\delta}\beta_{g_1} = 3g_1\partial_{g_\delta}\gamma_1$, one receives the implicit expressions for C_α :

$$C_\alpha = [\partial_{g_\alpha} - \frac{1}{3g_1}\delta_{\alpha 1}\cdot D_\nu]W.$$

One can make out them obviously:

$$\begin{array}{l} C_1 = (Z_1/2\varepsilon g_1 \cdot (\gamma_1 - 2\varepsilon/3))v'\Delta v + (Z_2/2\varepsilon g_1 \cdot (\gamma_2 - 2\varepsilon/3))\lambda\theta'\Delta\theta; \\ C_2 = v'\Delta v + \chi v'(\theta\partial)\theta, \quad \chi = (\partial_{g_2}Z_1)^{-1}Z_3{g_1}^{1/2}\nu^{1/2}M^{(2a-1)\varepsilon} \\ C_3 = \lambda(1+\lambda\partial_\lambda)Z_2\theta'\Delta\theta. \end{array}$$

(Here we have taken advantage by that in the first order of $\varepsilon \quad \gamma_i = 2\varepsilon g_1\partial_{g_1}Z_i$.)

The operators $v'\Delta v, \quad \theta'\Delta\theta$ and $v'(\theta\partial)\theta$ are contained in the action (11); dimensions of C_α are:

$$\Delta_{C_1} = 3\frac{2}{3} + \omega_1, \quad \Delta_{C_2} = 3\frac{2}{3} + \omega_2, \quad \Delta_{C_3} = 3\frac{2}{3} + \omega_3 \tag{12}$$

Essentiality of the contributions of particular combinations depends on a parity between the correction indexes ω_α and is defined by the least of them.

Renormalization of the operators ($N^0 1$, tab.1) proceeds in a class of tensors at a zero external momentum and frequency:

$$\mathcal{G} = \{\partial(v'v), \quad \partial(\theta'\theta), \quad v'_i\partial_j v_i, \quad \theta'_i\partial_j\theta_i\}.$$

The dimensions of two first operators of family $\mathcal{G}$ is $\Delta = 4$, as far as these operators don't renormalize and don't admix to any others, as a structure of interaction in the theory of type [1], [7] provides removal one derivative on each external line v' and θ' - types from each 1-irreducible diagram, that effectively lowers an index of divergence of the diagram.

Two last operators in $\mathcal{G}$ are the vectors not equal to zero at a zero external momentum. The operator $\mathcal{G}_1 = v'_i\partial_j v_i$ is finite. One can represent it in a kind of a difference of two finite operators: $\partial_j(v'_iv_i) - v_i\partial_j v'_i$. (The finiteness of $v_i\partial_j v'_i$ in the Galileian invariant theories is proven in [5]. This operator is Galileian invariant, therefore it can not admix to $\mathcal{G}_1$). By virtue of finiteness of $\mathcal{G}_1$, operator $\mathcal{G}_2 = \theta'_i\partial_j\theta_i$ does not admix to it.

As a whole, the matrix of renormalization constants has a kind of:

$$\mathbf{Z} = \begin{pmatrix} 1 & Z_{12} \\ 0 & Z_{22} \end{pmatrix}.$$

In one-loop order approach the unknown elements of matrix Z_{ik} are calculated under the diagrams listed on firure 1. It gives:

$$Z_{12} = \frac{\lambda(d+2(d-1)}{(\lambda+1}(\frac{g'_1}{\varepsilon} - \frac{g'_2}{a\varepsilon}), \quad Z_{22} = 1 - \frac{(d-1)(d+2}{2(\lambda+1}(\frac{g'_1}{\varepsilon} + \frac{g'_2}{a\varepsilon}). \tag{13}$$

With the known matrix Z_{ik} a matrix of critical dimensions $\Delta_{ik} = (d_F^k)_{ik} + \Delta_\omega (d_F^\omega)_{ik} + \gamma_{ik}$ is calculated. Critical dimensions of frequency in the incompressible theory [8] are $\Delta_\omega = 2/3$ in kinetic regime and $\Delta_\omega = 2$ for magnetic one. The concrete critical dimensions there are eigenvalues of matrix Δ_{ik}. They are corresponded to the linear combinations $U_{ik} F_k^R$ making Δ_{ik} diagonal. Critical dimensions belong to the operator $\mathcal{G}_2, \quad \Delta_{\mathcal{G}_2} = 5\frac{1}{3}$, as well as combination $\mathcal{G}_2 + \mu\mathcal{G}_1, \quad \Delta_{\mathcal{G}_2+\mu\mathcal{G}_1} = 4, \quad \mu = \gamma_{12}\gamma_{22}^{-1}$.

It is important to note that the scaling behaviour in the theory is determined by not renormilized composite operators. The relation between the renormalized combination and the not renormilized ones is followed:

$$F_\delta^R = U_{\delta\alpha} Z_{\alpha\beta}^{-1} F_\beta.$$

Thus we have

$$\mathcal{G}_1^R + \mu\mathcal{G}_2^R = \mathcal{G}_1 + (\mu - Z_{12}) Z_{22}^{-1} \mathcal{G}_2, \quad \mathcal{G}_2^R = Z_{22}^{-1} \mathcal{G}_2.$$

The most essential contribution connected with $\mathcal{G}_2$ is given by a linear combination of the operators $\mathcal{G}_1$ and $\mathcal{G}_2$; it has the dimension $\Delta = 4$ ($N^0 1$, tab. 1).

The dimension of some composite operators can be solved without calculating of the appropriate diagrams with the help of Schvinger equations. One can consider for example equality of a kind:

$$0 = \int \mathcal{D}\Phi \frac{\delta}{\delta v_i'(x)} \{ g_2 \theta^2(x) e^{S_R + A\Phi} \}.$$

In designations $< F >_A \equiv \frac{\int D\Phi F(x,\Phi) e^{S_R(\Phi)+A\Phi}}{\int D\Phi e^{S_R(\Phi)+A\Phi}}$ it will be recorded as

$$< G_2 \theta^2(x) V_i(x) >_A = -g_2 A_{v'}(x) < \theta^2(x) >_A, \tag{14}$$

where

$$V_i \equiv \frac{\delta S_R(\Phi)}{\delta v_i'(x)} = (d_{vv} v')_i + (d_{v\theta}\theta')_i - \mathcal{D}_t v_i + \nu Z_1 \Delta v_i + \\ + Z_3 g_2 g_1^{-1/2} \nu^{3/2} M^{(2a-1)\varepsilon} (\theta\partial)\theta_i - g_2 \nu^3 \partial_i \theta^2/2 + \partial_i F_p.$$

The dimension of object in a left-hand part of equality (14) is defined as follows:

$$\Delta_{FV_i} = \Delta_F - \Delta_{v'} - \Delta_t - \Delta_x, \quad F = g_2\theta^2. \tag{15}$$

It should be noticed, that the contribution of random forces in V_i

$$\theta^2 (d_{v\theta}\theta')_i + \theta^2 (d_{vv} v')_i \equiv \theta^2(x) \left(\int dx d_{ij}^{vv}(x-y) v_j'(y) + \int dy d_{ij}^{v\theta}(x-y) \theta_j'(y) \right)$$

is finite and doesn't admix to any other considered operators because of presence of closed cycles of retarded lines in 1-irreducible diagrams. Hence, it can be cancelled.

In a kinetic regime at the real value of ε the critical dimensions are $\Delta_x = -d$, $\Delta_t = -2/3$, $\Delta_{v'} = 3\frac{1}{3}$ [7]. Then, with the accounting of (15) one can receive for a combination $\mathcal{J}_1 = g_2\theta^2 V_i$:

$$\Delta_{\mathcal{J}1} = 2\frac{1}{3} - 4a + \omega_2.$$

An equation

$$0 = \int \mathcal{D}\Phi \frac{\delta}{\delta v_i'(x)}\{g_2 v'\theta^2(x)e^{S_R + A\Phi}\}, \tag{16}$$

is useful for a determination of dimension of a combination $\mathcal{J}_2 = v'\theta^2(-\mathcal{D}_t v + \nu\Delta v - g_2\nu^3\partial\theta^2/2 + \partial F_p + g_2 g_1^{-1/2}\nu^{3/2}(\theta\partial)\theta)$

With the preceding notation the equation has a kind of:

$$< g_2\theta^2(x)V_i(x) >_A + (d-1)g_2\delta^{(d+1}(x-y)|_{x=y}\theta^2(y) = -g_2 A_{v'}(x) < \theta^2(x) >_A .$$

Formally infinite term in a left-hand part of equality containing $\delta^{(d+1)}(0)$ will be cancelled with the similar ultraviolet divergent contribution on the average $< g_2\theta^2(x)V_i(x) >_A$; at calculations in the frame of network of minimal subtraction (MS) they can be simultaneously ignored. Then the formula (15) permits to receive the required dimension, if a dimension of the operator $X = g_2 v_i'\theta^2$ is known.

To study the dimension it is necessary to analyse renormalization of family of the operators:

$$\mathcal{X} = \ v_i'v^2, \quad g_2 v_i'\theta^2, \quad \theta_i'\theta v \ .$$

The operators of $\mathcal{X}$ don't mix with the operators of $\phi'\partial\phi$ - type, as far as the 1-irreducible diagrams do not contain divergences. The last operator of family $\mathcal{X}$ is not mixed with the other operators for the same reason. Furthermore the Galileian invariancy of the considered theory causes finiteness of the operators $v_i'v^2$, $\theta_i'\theta v$ ($\Delta = 4$) and the absence of admixing of $v_i'v^2$ to $v_i'\theta^2$.

It is obviously that the critical dimension inherent in kinetic mode to combination of the operators $\mathbf{X} = \mathcal{X}_1^R + \mu_3\mathcal{X}_2^R = \mathcal{X}_1 + (\mu_3 - Z_{12})Z_{22}^{-1}\mathcal{X}_2$, $\mu_3 = \gamma_{12}\gamma_{22}^{-1}$ is $\Delta_{\mathbf{X}} = 2\frac{2}{3}$. Independently the operator $\mathcal{X}_2^R = Z_{22}^{-1}\mathcal{X}_2$ has a critical dimension too. Appropriate constants Z_{12} and Z_{22} (determining the anomalous γ_{ik}) in one-loop approach can be calculated on diagrams located on figure 2.

$$Z_{12} = -\frac{\lambda(d+2)(d-1)}{2(\lambda+1)}(\frac{g_1'}{\varepsilon} - \frac{g_2'}{a\varepsilon}),$$

$$Z_{22}Z_\theta^2 = 1 + \frac{(d-1)(d+2)}{2(\lambda+1)}(\frac{g_1'}{\varepsilon} - \frac{g_2'}{a\varepsilon}).$$

So for critical dimension of $\mathcal{X}_2$ at the real values of ε and d one can receive:

$$\Delta_{\mathcal{X}_2} = 6\frac{2}{3} - 4a.$$

The most essential contribution, connected with the operator $g_2 v_i' \theta^2$ is $\Delta = 2\frac{2}{3} + \omega_2$ (at $a < 1$) (stipulated by a combination $\mathbf{X}$) and $\Delta = 6\frac{2}{3} - 4a + \omega_2$ (in the case of $a > 1$) (when the contribution of $\mathcal{X}_2^R$ becomes essential).

In view of this result from the formula (15) is received scaling dimension for the combination $\mathcal{J}_2$:

$$\Delta_{g_2 v_i' \theta^2 V_i} = \begin{cases} 5\frac{2}{3} - 4a + \omega_2, & a > 1 \\ 1\frac{2}{3} + \omega_2, & a < 1. \end{cases}$$

5 Results and Discussion concerned with the kinetic regime

A dimensionless variable $k^2 c^2/\omega^2$ in (11) defines the scaling dimension of c as $\Delta_c = -1/3$, as well as in the theory of ordinary compressible fluid [5]. Then with the propagators (11) it is possible to determine of scaling dimension of longitudinal and scalar fields:

$$\Delta_p = -2/3, \quad \Delta_{p'} = 3\frac{2}{3}, \quad \Delta_u = -1, \quad \Delta_{u'} = 3\frac{1}{3}. \tag{17}$$

In the framework of Landau ideas to get an "effective" functional of action the developed turbulent spectra in the kinetic mode are described, one should reject all the unessential operators. Problem about whether the operator is essential or not we decide proceeding from its scaling dimension calculated at real values of ε and d. Results of comparisons of dimensions of the operators are shown in tab. 2.

There are the operators having the same order on c^{-2} and connected in functional (10) with identical sets of longitudinal and scalar fields are compared. In the first column the numbers of lines of table 1 are specified, where the essential operators of smaller dimension are located; in the second column the unessential operators are displayed. The conditions at which comparison of the operators is executed are indicated in the third column of the table.

The analysis of data of table 2 shows that depending on size of parameter a whether the composite operators containing the not local insertion of F_p-type or the operators constructed with participation of local insertion $g_2\theta^2$ (9) are essential. The boundary value of a at which shift of a critical mode occurs is $a = 2/3 + \omega_2/4 = 5/6 - 1/8\gamma \simeq 0.91$. As was already specified down to $a \geq 1.1$ the leading contribution in F_p is connected with the combination $\mathcal{R}_2$ Thus, the contribution of $\partial_i\theta_k\partial_k\theta_i$ in kinetic regime of the model are correction always.

To get the "effective" functional of action depending on value of parameter a we should exclude the various unessential operators. Resulting functional has a form

of:

$$
\begin{gathered}
S(\Phi) = 1/2 v_i' d_{ij}^{vv} v_j' + 1/2 \theta_i' d_{ij}^{\theta\theta} \theta_j' + \xi v_i' d_{ij}^{v\theta} \theta_j' + 1/2 u_i' D_{ij}^{uu} u_j' + v_i'[-\mathcal{D}_t v_i - \\
-1/c^2 \left((v\partial)\tilde{u}_i + (\tilde{u}\partial) v_i)\right] + \theta_i'[-\mathcal{D}_t \theta_i + \lambda\nu\Delta\theta_i - g_1 \nu^{3/2} (\theta\partial) v_i + 1/c^2((\theta\partial)\tilde{u}_i + \\
+\nu\Delta v_i - \theta_i(\partial\tilde{u})] + u_i'[-1/c^2 \mathcal{D}_t \tilde{u}_1 / c^2 g_1^{1/2} \nu^{3/2} (v\partial)\tilde{u}_i + \nu'/c^2 \Delta u_i + (\tilde{u}\partial) v_i - \\
-1/c^2 \nu' \partial_i \partial_t q + 1/c^2 \Delta^{-1} \partial_i \partial_t^2 q - 1/c^4 (\tilde{u}\partial)\tilde{u}_i - \partial_i p + \\
+1/2(1 + \tilde{q}/c^2) \cdot 1/c^2 \partial_i \tilde{q}^2] - p'\Big[(\partial u + 1/c^2 \partial_i(\tilde{q}\tilde{u}_i)\Big],
\end{gathered}
\tag{18}
$$

where we have used the notations followed:

$$
Q \to p + g_2 \nu^3 \Theta(a - 2/3 - 1/4\omega_2)\theta^2/2, \quad \tilde{q} = q - F_p, \quad \tilde{u}_i = u_i + F_{1i} - F_{2i} q, \tag{19}
$$

and $F_p = g_1 \nu^3 \Delta^{-1} \Theta(2/3 + 1/4\omega_2 - a) \partial_i v_k \partial_k v_i$, $\quad F_{1i} = \Delta^{-1} \partial_i \mathcal{D}_t F_p$, $\quad F_{2i} = \Delta^{-1} \partial_i \mathcal{D}_t$, and $\Theta(x)$ is theta function.

The amendment of the first order on c^{-2} to spectra of developed turbulence in kinetic regime in the area of $a > 2/3 + \omega_2/4$ is given by the operator $\mathcal{D}_t F_2 \theta^2$ ($N^0 30$, tab. 3): $\sim c^{-2} k^{-2(2a - \omega_2/2 - 1)}$.

We have defined the scaling dimension of c as $\Delta_c = -1/3$. Now it is visible that in accordance with results of the theory in the framework of approach of one insertion of the composite operators of transversal fields our estimation is fair at $a < 2/3 + \omega_2/4$. In case of strong singularity of magnetic pumping of energy the scaling behaviour of c becomes more essential $\sim k^{1+\omega_2/2-2a}$. The dimension of pressure field p thus should be evaluated on an insertion of operator $g_2\theta^2/2$, $\quad \Delta_p = 2 + 1/2\omega_2 - 4a$. And we have:

$$
\Delta_{p'} = 1 + 4a - \omega_2, \quad \Delta_u = 1\frac{2}{3} - 4a + \omega_2, \quad \Delta_{u'} = \frac{2}{3} + 4a - \omega_2. \tag{20}
$$

The new variable appearing thus in the inertial range is proportional $Ma\ k^{1-2a+\omega_2/2}$. The amendments to spectra of developed turbulence due to compressibility of a fluid are essential in the scaling area of the spectrum of scales.

We shall notice that the results yielded are received by us in approach of one insertion of operators of transversal fields, so it would be changed in the case of accounting of more than one insertion.

6 On the IR perturbation theory in magnetic regime

In the magnetic fixed point (7) all the invariant charges approaches to zero. For the formulation the nontrivial perturbation theory it needs to be redefined fields, parameters, and the variable of time:

$$
\begin{gathered}
\theta \to \sqrt{\lambda}\theta, \quad \theta' \to 1/\sqrt{\lambda}\theta', \quad v \to \lambda v, \quad v' \to v', \quad t \to t/\lambda, \\
u \to \sqrt{\lambda} u, \quad P \to \sqrt{\lambda} p, \quad u' \to \sqrt{\lambda} u', \quad p' \to \sqrt{\lambda} p', \quad c^{-2} \to \sqrt{\lambda} c^{-2}.
\end{gathered}
\tag{21}
$$

We shall note, that the redefinition of transversal fields in (21) doesn't change a position of the magnetic fixed point (7) in the theory of incompressible fluid, [8].

The action functional agreed with decomposition on c^{-2} with the preceding notation $\tilde{q}$, $\tilde{u}$ will be recorded as

$$
\begin{aligned}
&S(\Phi) = 1/2Bg_1'\nu^3 v_i' d_{ij}^{vv} v_j' + 1/2Bg_2'\nu^3 \theta_i' d_{ij}^{\theta\theta}\theta_j' + B\xi v_i' d_{ij}^{v\theta}\theta_j' + 1/2\lambda u_i' D_{ij}^{uu} u_j' + v_i'\Big[-\lambda\partial_t v_i - \\
&-\lambda(v\partial)v_i - 1/c^2\lambda\left((v\partial)\tilde{u}_i + (\tilde{u}\partial)v_i\right) + 1/(1+\lambda\tilde{q}/c^2)\Big(\nu\Delta v_i + (\theta\partial)\theta_i + 1/c^4\lambda\tilde{q}\Delta\tilde{u}_i\Big)\Big] + \\
&+\theta_i'\Big[-\partial_t\theta_i + \lambda\nu\Delta\theta_i - \left((v\partial)\theta_i + (\theta\partial)v_i\right) + 1/c^2\left((\theta\partial)\tilde{u}_i - \theta_i(\partial\tilde{u} - (\tilde{u}\partial)\theta_i\right)\Big] + \\
&+u_i'[-1/c^2\lambda^{3/2}\partial_t\tilde{u}_1/c^2\lambda^{3/2}\left((v\partial)\tilde{u}_i + (\tilde{u}\partial)v_i\right) - 1/c^4\lambda^{3/2}(\tilde{u}\partial)\tilde{u}_i - \partial_i p + \\
&+1/(1+\lambda\tilde{q}/c^2)(-1/c^2\lambda\nu\tilde{q}\Delta v_i + 1/c^2\lambda^{1/2}\nu'\Delta\tilde{u}_i - 1/2\cdot 1/c^2\lambda\partial_i\tilde{q}^2 - \\
&-1/c^2\lambda^{3/2}\tilde{q}(1/2\partial_i\theta^2 - (\theta\partial)\theta_i))] - p'\Big[(\partial u + 1/c^2\lambda\partial_i(\tilde{q}\tilde{u}_i)\Big].
\end{aligned}
\tag{22}
$$

As the operators of shift for $\tilde{q}$, $\tilde{u}$ in (22) are chosen $F_p = \lambda^{1/2}\Delta^{-1}\partial_i\partial_k$ $(\lambda v_i v_k - \theta_i\theta_k)$, $\quad F_{1i} = \lambda\Delta^{-1}\partial_i\mathcal{D}_t F_p$, $\quad F_{2i} = \lambda\Delta^{-1}\partial_i\mathcal{D}_t$.

The square-law part of (22) defines the propagators functions in a kind of:

$$
\begin{gathered}
G^{vv'} = \left(G^{v'v}\right)^+ = 1/\lambda_0 L_1^{-1}, \quad G^{\theta\theta'} = \left(G^{\theta'\theta}\right)^+ = L_2^{-1}, \quad G^{vv} = G^{vv'}D_{vv}G^{v'v}, \\
G^{\theta\theta} = G^{\theta\theta'}D_{\theta\theta}G^{\theta'\theta}, \quad G^{uu'} = \left(G^{u'u}\right)^+ = \omega L_3^{-1}, \quad G^{p'p} = \left(G^{qp'}\right)^+ = -(i\omega + \nu' k^2/\lambda)L_3^{-1}, \\
G^{uu} = G^{uu'}D_{uu}G^{u'u}, \quad G^{pp} = G^{pu'}D_{uu}G^{u'p}, \quad G^{up} = 1/c^2 G^{uu'}D_{uu}G^{u'p}, \\
G^{p'u} = i\mathbf{k}/(\lambda k^2), \quad G^{u'q} = i\mathbf{k}L_3^{*-1}/\lambda, \\
L_1 = -i\lambda_0\omega + \nu_0 k^2, \quad L_2 = -i\lambda_0\omega + \lambda_0\nu_0 k^2, \quad L_3 = -i\lambda\omega + \nu' k^2 + ik^2c^2/(\lambda\omega)
\end{gathered}
\tag{23}
$$

The propagators of transversal fields are multiple to P_{ij}; other ones have the factor of Q_{ij}.

The singularities of functions (23) on $1/\lambda$ are displayed in the δ-functions at $\lambda \to 0$. However, the infringement of correctness of the perturbation theory does not occur as far as all the in (22), except $\theta_i'\left((\theta\partial)\tilde{u}_i - \theta_i(\partial\tilde{u}) - (\tilde{u}\partial)\theta_i\right)$, contain in factors the positive degrees of parameter λ. Having seen the diagrams before passage to the limit $\lambda \to 0$ it is easy to be convinced that all the singularities in them are eliminated by integration on a time in the vertices and are reduced by factors at interactions.

The graphs constructed with participation of vertices $\theta_i'\left((\theta\partial)\tilde{u}_i - \theta_i(\partial\tilde{u}) - (\tilde{u}\partial)\theta_i\right)$ contain functions of propagators independent from λ, they haven't the singularities of specified type.

In the expansion series for multipliers $1/(1+\lambda\tilde{q}/c^2)$ in the functional (22) there are the vertices with a number of scalar fields p as well as with insertions of the composite operators of transversal fields F_p, $\lambda\theta^2$. $\tilde{q}$ participates in these decompositions in a combination with parameter λ. Thereof it is possible to assert that appropriate singularities are absent in all the orders of the perturbation theory.

As follows from [8] in a magnetic regime in the theory of incompressible fluid $\gamma_{\nu*} = 0$. Therefore in a magnetic mode the invariant variable of frequency $\bar{\nu}k^2/\omega$ defines the dimension of ω as $\Delta_\omega = 2$. The behaviour of ν' in the scaling area is not known; assuming ν' as scaling dimensionless parameter, we receive $\Delta_\omega = 2$ also. Variable ck/ω in this case permits to evaluate the dimension of c as $\Delta_c = 1$. Then the amendments due to compressibility to spectra of turbulence in the scaling area appear unessential in magnetic regime.

To justify this assumption it is necessary to find out, what scaling dimension does the parameter c^{-2} demonstrate in the diagrammatic contributions of the perturbation theory. It is reasonable to consider the leading scaling asymptotics without the accounting of correction indexes ω_α.

In this case after fulfillment of functional integration of $G(A_\phi)$ on the transversal fields the set of diagrams of considered perturbation theory will be consisted of the graphs with the lines of longitudinal and scalar fields adjoined to the composite operators of transversal fields located in the table 3.

Renormalization of the operators contained in the table had been discussed by us. The dimension ($N^0 1$, tab. 3) is inherent to the operator $\theta'\partial\theta$ in the combination $\theta'\partial\theta + \lambda\mu_3 v'\partial v$, where $\lambda\mu_3 = \mu$. As far as formally in the magnetic point $\lambda_* = 0$, the contribution of the hydrodynamic operator is absent. (With the accounting of (13) in magnetic point (7) the eigen dimension of operator $\theta'\partial\theta$ is equal to $4 + 20a$, so the appropriate contribution is unessential.)

Thus, the leading scaling asymptotics in the theory of developed turbulence of a compressible conductive fluid in a magnetic regime is given by the action functional as follows:

$$\begin{aligned} S(\Phi) = {} & 1/2 v'_i D^{vv}_{ij} v'_j + 1/2 \theta'_i D^{\theta\theta}_{ij} \theta'_j + \xi v'_i D^{v\theta}_{ij} \theta'_j + 1/2 u'_i D^{uu}_{ij} u'_j + v'_i \left(-\lambda \partial_t v_i + \nu \Delta v_i\right) + \\ & + u'_i (-1/c^2 \cdot \lambda^{3/2} \partial_t u_i + \lambda^{1/2} \nu'/c^2 \Delta u_i - \partial_i q - 1/c^2 \cdot \lambda \nu' \partial_i \partial_t q + 1/c^2 \cdot \lambda^2 \Delta^{-1} \partial_i \partial_t^2 q) - \\ & - p'(\partial u + \theta'_i(-\partial_t \theta_i + \nu \Delta \theta_i - ((v\partial)\theta_(\theta\partial) v_i) + 1/c^2((\theta\partial)\tilde{u}_i - \theta_i(\partial \tilde{u} - (\tilde{u}\partial)\theta_i)) \end{aligned} \tag{24}$$

After renormalization of transversal fields in (24) for appropriate generating functional of correlation functions it is possible to develop the standard perturbation theory on formally small parameter c^{-2} for Green functions that are the intrinsic tensors with distinguished arguments of time. In the contribution of each order of the perturbation theory the finite number of diagrams are participated. For construction of decomposition for the functions, containing odd number of fields θ as well as functions not integrated on time, it is necessary to consider the theory with respect of the correction indexes. As is shown in [9], the scaling representations for such functions contain an additional small factor of $\mathcal{O}(s^{\omega_\alpha})$.

The main amendments to Green functions in the theory of $\simeq c^{-4}k^4$ that justifies the assumption stated above about critical dimensions $\Delta_c = 1$ and $\Delta_t = -2$, and about the amendments due to compressibility of fluid are unessential. Formally these amendments are presented in a kind of power series of parameter $Ma\, k$. In the contrast with the kinetic regime here at any value of Ma the parameter connected with the compressibility does not penetrate into the inertial range and does not essentially influence on turbulent behaviour in magnetic hydrodynamics.

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References

[1] De Dominicis C. Martin P. C. *Phys Rev A* , 1979, v. **29** , p. 419.

[2] Adjemian L.Ts., Vasil'ev A.N., Pis'mak Yu.M. *Teor. and mat. Physics* , 1983, v. **57** , N2, p. 268.

Adjemian L.Ts., Antonov N.V., Vasil'ev A.N. *the Journal of exp. and teor. Physics* , 1989, v. **95** (4), p. 1272.

[3] Antonov N.V. *Vestnik St.-Petersburg Univ.* ser. 4: Physics, chemistry. 1992, v.4, p. 25

[4] Adjemian L.Ts., Nalimov M.Yu., Stepanova M.M. *Teor. and mat. Physics* **v.104**, $N^0$2, 1995, pp. 260-270.

[5] Wolchenkov D.Yu., Nalimov M.Yu. *Teor. and mat. Physics* , 1995, **v.106** ,$N^0$3, pp. 375-389.

[6] Fournier J.-P., Frish U. *Phys. Rev.*,1983. Vol. A28, N2. P.1000.

[7] Adjemian L.Ts. , Vasil'ev A.N., Gnatich M. *Teor. and mat. Physics* , 1985, **v. 64** , N2, p. 196.

[8] Adjemian L.Ts., Wolchenkov D.Yu., Nalimov M.Yu. *Teor. and mat. Physics* , 1995, **v. 107** ,$N^0$1, page 142-154.

[9] Wolchenkov D.Yu. **chao-dyn/9607009**

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Table 1

Composite operators of transversal fields				
N.	**Operator**	**Dimension**	**Fields**	**Factor**
1	$v'\partial v, \quad \theta'\partial\theta$	4	u	$1/c^2$
2	$\Delta F_1 {F_p}^l$	$1-2l/3$	$u', \quad q^n$	$1/c^{2(l+n+1)}$
3	$v'\Delta v {F_p}^l$	$3\frac{2}{3}-2l/3+\omega_{1,2}$	q^n	$1/c^{2(l+n)}$
4	$\theta'\Delta\theta {F_p}^l$	$3\frac{2}{3}-2l/3+\omega_{1,3}$	q^n	$1/c^{2(l+n)}$
5	$\Delta F_2 {F_p}^l$	$1\frac{2}{3}-2l/3$	$u', \quad q^{n+1}$	$1/c^{2(l+n+1)}$
6	$v'\Delta F_1 {F_p}^l, \quad (l+n)>0$	$4\frac{1}{3}-2l/3$	q^n	$1/c^{2(l+n+1)}$
7	$\partial {F_p}^{l+1}$	$1-2(l+1)/3$	$u', \quad q^n$	$1/c^{2(l+n)}$
8	$v'\Delta {F_p}^l, \quad (l+n)>0$	$5\frac{1}{3}-2l/3$	$u, \quad q^n$	$1/c^{2(l+n+1)}$
9	∂F_{v1}	0	$p', \quad q$	$1/c^2$
10	$v'\Delta F_2 {F_p}^l, \quad (l+n)>0$	$5-2l/3$	q^{n+1}	$1/c^{2(l+n+1)}$
11	∂F_2	$2/3$	$p', \quad q^2$	$1/c^2$
12	$\mathcal{D}_t F_1$	$-1/3$	u'	$1/c^2$
13	∂F_p	$1/3$	$u, \quad p'$	$1/c^2$
14	$F_1\partial v$	$-1/3$	u'	$1/c^2$
15	$F_1\partial F_p$	$-2/3$	p'	$1/c^2$
16	$\mathcal{D}_t F_2$	$1/3$	$q, \quad u'$	$1/c^2$
17	$F_2\partial F_p$	0	$q, \quad p'$	$1/c^2$
18	$F_2\partial v$	$1/3$	$q, \quad u'$	$1/c^2$
19	∂F_1	0	$u, \quad u'$	$1/c^4$
20	∂v	$2/3$	$u, \quad u'$	$1/c^2$
21	∂F_2	$2/3$	$q, \quad u, \quad u'$	$1/c^4$
22	$F_2\partial F_1$	$-1/3$	u'	$1/c^4$
23	$\Delta {F_p}^l$	$2-2l/3$	$u, \quad q^n, \quad u'$	$1/c^{2(n+l+1)}$
24	$F_1\partial F_2$	$-1/3$	$q, \quad u'$	$1/c^4$
25	$v'\partial v F_2, \quad \theta'\partial\theta F_2$	$3\frac{2}{3}$	q	$1/c^2$
26	$F_2\partial F_2$	$1/3$	$u', \quad q^2$	$1/c^4$
27	$v'\partial v F_1, \quad \theta'\partial\theta F_1$	3	-	$1/c^2$
28	$\Delta v {F_p}^l, \quad (l+n)>0$	$1\frac{2}{3}-2l/3$	$u', \quad q^n$	$1/c^{2(l+n)}$
29	$v'(\theta\partial)\theta F_p$	$3+\omega_{1,2}+\omega_2$	q^n	$1/c^{2(n+1)}$
30	$\mathcal{D}_t F_2\theta^2$	$2\frac{1}{3}-4a+\omega_2$	u'	$1/c^2$
31	$\theta^2\mathcal{D}_t v$	$2\frac{1}{3}-4a+\omega_2$	$u', \quad q^n$	$1/c^{2(n+1)}$
32	$\theta^2\Delta v$	$2\frac{1}{3}-4a+\omega_2$	$u', \quad q^n$	$1/c^{2(n+1)}$
33	$\theta^2(\theta\partial)\theta$	$2\frac{1}{3}-4a+\omega_2$	$u', \quad q^n$	$1/c^{2(n+1)}$
34	$\theta^2\Delta$	$4-4a+\omega_2$	$u', \quad u, \quad q^n$	$1/c^{2(n+1)}$
35	$v'\theta^2\mathcal{D}_t v,\ a>1$	$5\frac{2}{3}-4a+\omega_2$	q^n	$1/c^{2(n+1)}$
35a	$v'\theta^2\mathcal{D}_t v,\ a<1$	$1\frac{2}{3}-4a+\omega_2$	q^n	$1/c^{2(n+1)}$
36	$v'\theta^2\Delta v,\ a>1$	$5\frac{2}{3}-4a+\omega_2$	q^n	$1/c^{2(n+1)}$
36a	$v'\theta^2\Delta v,\ a<1$	$1\frac{2}{3}-4a+\omega_2$	q^n	$1/c^{2(n+1)}$
37	$v'\theta^2(\theta\partial)\theta,\ a<1$	$5\frac{2}{3}-4a+2\omega_2$	q^n	$1/c^{2(n+1)}$
37a	$v'\theta^2(\theta\partial)\theta,\ a>1$	$1\frac{2}{3}-4a+2\omega_2$	q^n	$1/c^{2(n+1)}$
38	$\theta^2\partial\mathcal{D}_t F_p$	$3-4a+\omega_2$	$u', \quad q^n$	$1/c^{2(n+1)}$
39	$v'\theta^2\Delta,\ a>1$	$8\frac{2}{3}-4a+\omega_2$	$u, \quad q^n$	$1/c^{2(n+2)}$
39a	$v'\theta^2\Delta,\ a<1$	$4\frac{2}{3}+\omega_2$	$u, \quad q^n$	$1/c^{2(n+2)}$
40	$v'\theta^2\Delta F_1,\ a>1$	$7\frac{2}{3}-4a+\omega_2$	q^n	$1/c^{2(n+1)}$
40a	$v'\theta^2\Delta F_1,\ a<1$	$3\frac{2}{3}+\omega_2$	q^n	$1/c^{2(n+1)}$
41	$\theta^2\partial F_2$	$2\frac{2}{3}-4a+\omega_2$	$u', \quad u$	$1/c^4$
42	$\partial\mathcal{D}_t\theta^2$	$3\frac{2}{3}-4a+\omega_2$	u'	$1/c^2$
43	$\partial\theta^2$	$3-4a+\omega_2$	$u', \quad q$	$1/c^2$
44	$\theta'\theta\partial F_2\theta^2$	$5\frac{2}{3}-4a+\omega_2$	$-$	$1/c^2$

Table 2

COMPARISON OF OPERATORS OF TRANSVERSAL FIELDS			
N.	**Essential operators**	**Unessential operators**	**Comments**
1	$NN^0 3, 4$	$NN^0 6, 10$	$-$
2	$N^0 25$	$NN^0 3, 4$	$n = 1$
3	$N^0 1$	$NN^0 8$	$n = 0$
4	$N^0 12$	$NN^0 2, 28$	$n = 0$
5	$N^0 16$	$N^0 5$	$-$
6	$N^0 19$	$N^0 23$	$n = 0, \quad l = 1$
7	$NN^0 12, 14$	$NN^0 30$	$a < 2/3 + 1/4\omega_2$
8	$N^0 30$	$NN^0 12, 14$	$a > 2/3 + 1/4\omega_2$
9	$NN^0 16, 18$	$N^0 42$	$a < 2/3 + 1/4\omega_2$
10	$N^0 42$	$NN^0 16, 18$	$a > 2/3 + 1/4\omega_2$
11	$N^0 19$	$N^0 41$	$a < 2/3 + 1/4\omega_2$
12	$N^0 41$	$N^0 19$	$a > 2/3 + 1/4\omega_2$
13	$N^0 21$	$N^0 32$	$l = 1, \quad a < 2/3 + 1/4\omega_2$
14	$N^0 32$	$N^0 21$	$l = 1, \quad a > 2/3 + 1/4\omega_2$
15	$N^0 24$	$N^0 31$	$n = 1, \quad a < 2/3 + 1/4\omega_2$
16	$N^0 31$	$N^0 24$	$n = 1, \quad a > 2/3 + 1/4\omega_2$
17	$N^0 31$	$N^0 34$	$-$
18	$N^0 1$	$N^0 39, 39a$	$n = 0$
19	$N^0 27$	$N^0 43$	$a < 2/3 + 1/4\omega_2$
20	$N^0 43$	$N^0 27$	$a > 2/3 + 1/4\omega_2$
21	$N^0 25$	$N^0 29$	$-$
22	$N^0 35a - 36a$	$N^0 40$	$-$
23	$N^0 35a - 36a$	$N^0 37(-a)$	$-$

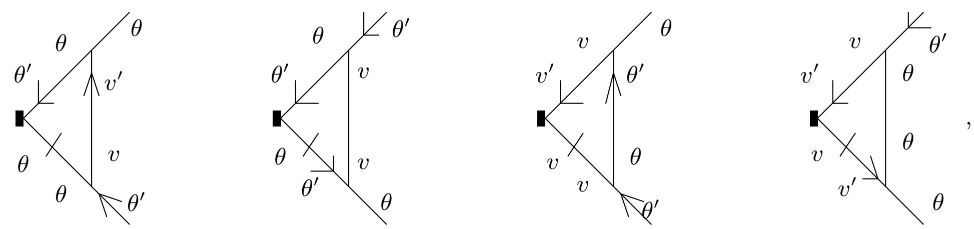

figure 1. The diagrams for renormalization of family of operators of $\partial\phi'\phi$ -type.

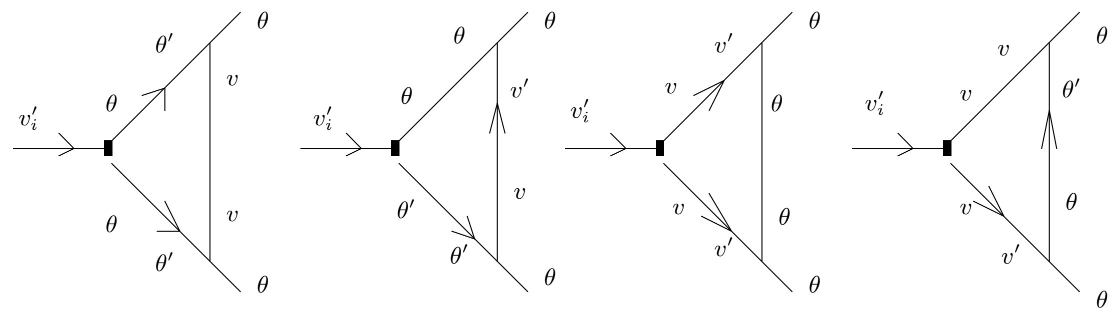

figure 2. The diagrams for renormalization of family of operators of $\phi'\phi^2$ -type.